Fermilab MTF

Doc Number:	MTF-94-004
Version:	
Category:	NOTE

Measurement of Harmonic Amplitudes and Phases Using Rotating Coils

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3/9/94; revised 1/19/98

1 Introduction

This report describes the procedure in which harmonic coefficients are extracted from rotating coil data at MTF. It is well known[1] that a two dimensional magnetic field may be represented by an analytic function and expanded about a point according to

$$B(r,\theta) = B_y + iB_x = \sum_{j=1}^{\infty} C_j r^{j-1} e^{i((j-1)\theta+\chi_j)} \tag{1}$$

In this equation, the coefficients C_j are the harmonic amplitudes and χ_j are the harmonic phases. The indices j use the convention that $j = 1$ represents the dipole harmonic, $j = 2$ the quadrupole, and so on. The units of C_j are in Tesla/m^{j-1}.

Most magnets measured at MTF are designed such that one harmonic, C_n, is dominant (e.g., $n = 1$ for dipole magnets). In these cases it is useful

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to express the harmonic amplitudes in dimensionless quantities, c_j, called normalized harmonics:

$$c_j = (C_j/C_n)\, r_0^{j-n} \tag{2}$$

The reference radius, r_0, is an arbitrary choice, but is usually chosen to be either the probe radius or some standard value (e.g., 2.54 cm). It is also useful to express the harmonic phases relative to χ_n, that is, in a coordinate system such that $\chi_n = 0$.

Alternatively, it is often useful to express the harmonics in terms of normal (b_j) and skew (a_j) components. These are defined by

$$b_j + i a_j = c_j e^{i\chi_j} \tag{3}$$

2 Flux in Rotating Coils

The generic coil used by MTF is wound on a cylinder of length L and radius r. Looking at a cross section of a coil in Figure 1a, the coil intersects the page at N vertices, where N must be an even number in order to make a complete coil. The complex coordinate of vertex k is $z_k = r e^{i\phi_k}$. If more than a single wire is used (e.g., Litz wire), then there will be M wires crossing at each vertex. The wires are connected at the ends of the cylinder so that there is a single conducting path through the entire coil. When the loop is rotated, a voltage is induced by the flux change in accordance with Faraday's Law. The induced voltage causes a current to flow through the loop. A direction is assigned to each vertex by following the current path through the loop, with positive direction for current flowing out of the page.

The flux measured by the coil as a function of rotation angle θ can be expressed in terms of the complex potentials at each vertex of the coil[2]:

$$\Phi(\theta) = -L\Re \sum_{j=1}^{\infty} \left(C_j e^{i\chi_j}\right) \eta_j e^{ij\theta} \tag{4}$$

The quantity η_j, called the *probe sensitivity*, contains all the factors due to the geometry of the probe. It is equal to

$$\eta_j = \frac{M}{j} r^j \sum_{k=1}^{N} \varepsilon_k e^{ij\phi_k} \tag{5}$$

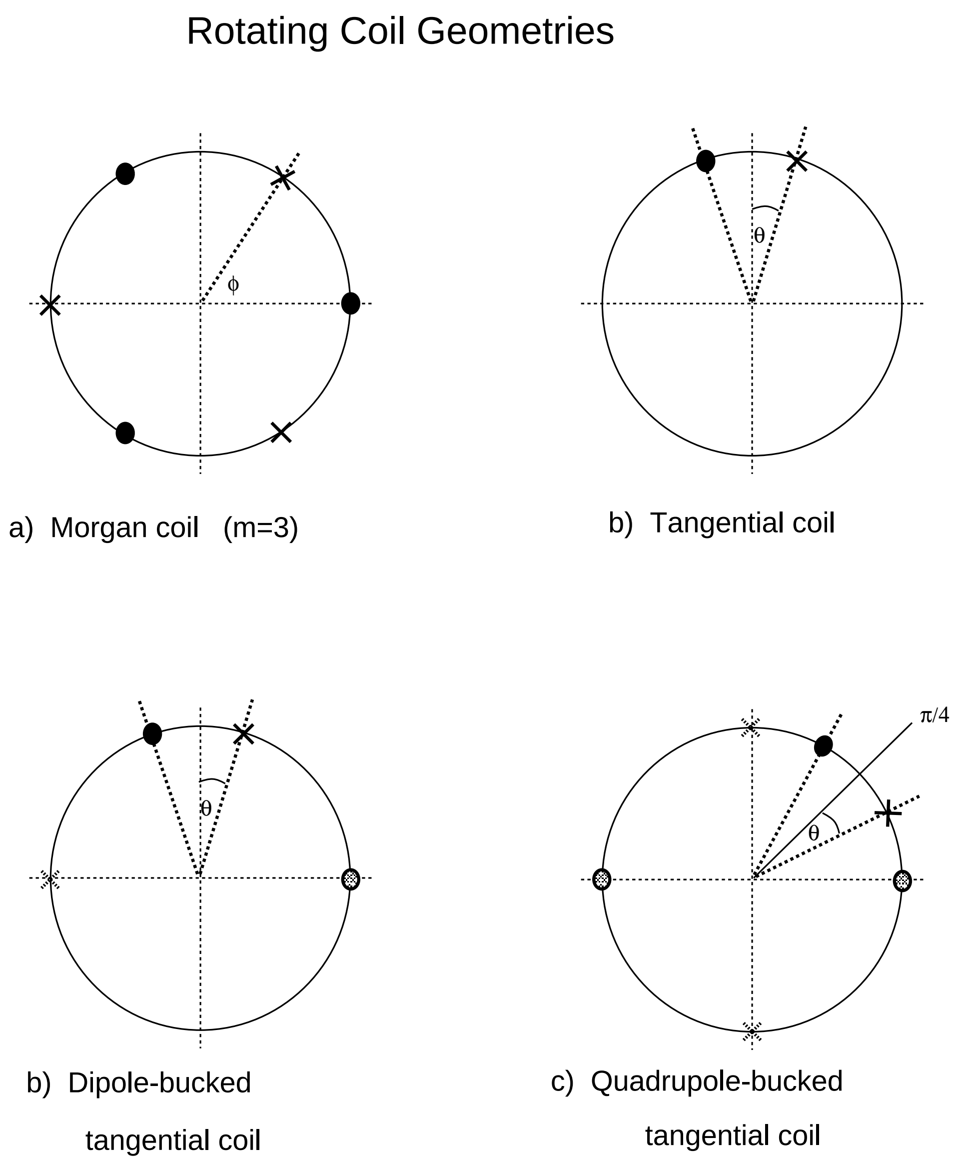

Figure 1: Cross sections of typical rotating coils. The dots indicate vertices with positive ε_j; crosses indicate vertices having negative ε_j.

where ε_k is the direction of vertex k (either $+1$ or -1). For convenience, we define the *probe geometry factor*, γ_j to be

$$\gamma_j = \sum_{k=1}^{N} \varepsilon_k e^{ij\phi_k} = g_j e^{i\xi_j} \tag{6}$$

Using these definitions, Equation 4 becomes

$$\Phi(\theta) = -\Re \sum_{j=1}^{\infty} (LC_j) \left(\frac{M}{j} r^j g_j \right) e^{i(\chi_j + \xi_j + j\theta)} \tag{7}$$

Substituting Eq. 2 to express the flux in terms of normalized harmonics, we have

$$\Phi(\theta) = -\Re \sum_j \left(A_n r_0^{n-j} \right) c_j \left(\frac{M}{j} r^j g_j \right) e^{i(\chi_j + \xi_j + j\theta)} \tag{8}$$

where we have defined $A_n = LC_n$ as the integrated reference amplitude. An additional simplification occurs if we make the "natural" choice for the reference radius r_0 by setting it equal to the probe radius:

$$\Phi(\theta) = \Re \sum_j \left(\frac{M}{j} A_n r^n c_j g_j \right) e^{i(\chi_j + \xi_j + \pi + j\theta)} \tag{9}$$

In the above, the $-$ sign is absorbed into the exponential as an additional factor of $i\pi$.

3 Fourier Transforms and Harmonic Coefficients

We collect an array of fluxes, $\Phi(\theta)$, on K uniformly spaced points in θ, and perform a Fourier Transform (the FFT code requires that K be a power of 2). The FFT returns the Fourier coefficients $F_j = \Phi_j e^{i\psi_j}$, which allow us to expand $\Phi(\theta)$ as

$$\Phi(\theta) = \Re \sum_{j=1}^{K/2} \Phi_j e^{i(\psi_j + j\theta)} \tag{10}$$

Comparing Equations 9 and 10, the harmonic coefficients can be related to the Fourier coefficients by equating the amplitudes and phases term by term:

$$\Phi_j = \frac{M}{j} A_n r^n c_j g_j \tag{11}$$

and

$$\psi_j = \chi_j + \xi_j + \pi \tag{12}$$

Note that the Φ_j are fluxes in units of T-m^2 (alternatively, in volt-seconds; the two units are equivalent). If the data were acquired with a V/f system, no conversion is necessary; but if an analog integrator was used, one must take into account the integrator time constant, probe resistance, and (if applicable) amplifier gain to convert the data to the correct units.

3.1 Reference amplitude

From Equation 11, and noting that by definition, $c_n = 1$, we see that the reference amplitude is equal to

$$A_n = \frac{n\Phi_n}{Mg_n r^n} \tag{13}$$

The reference phase, χ_n, is defined to be zero, but we need the observed Fourier phase, ψ_n, in order to determine the phases of the remaining harmonics.

3.2 Harmonic amplitudes

One could, in principle, obtain all the harmonics with a single coil, but this is not a good idea in practice, because the dominant harmonic is generally so much larger than the others. Instead, we use a reference coil to measure the reference amplitude, and then follow this with a flux measurement with a so-called "harmonics" coil that is insensitive, within manufacturing tolerances, to the reference harmonic. The normalized harmonic amplitudes are determined from the Fourier amplitudes of this harmonics run using Equation 11:

$$c_j = \frac{j\Phi_j}{MA_n r^n g_j} \tag{14}$$

For some types of coils, particularly Morgan coils, there will be values of j for which $g_j = 0$. In those cases, c_j is indeterminate and cannot be measured by that coil.

3.3 Harmonic phases

The phases are not as easy to calculate. We follow methods outlined in [3] and [2]. Equation 4 assumes the η_j coefficients have been calculated with the probe oriented at $\theta = 0$; however, the probe rotation is usually started at an angle which is not precisely determined. Also, since the reference coil and harmonics coil(s) are wound on the same probe cylinder, the relative orientations of each of these coils with respect to the starting point are different. These angular offsets must all be taken into account.

Let's define the offset angle for the reference coil as ϕ_r and the offset angle for the harmonics coil as ϕ_h. This offset is measured with respect to an arbitrary nominal orientation, since it will be seen that only the relative offset $\phi_h - \phi_r$ is important. Under a rotation ϕ, the probe vertices z_k are transformed to $z_k e^{i\phi}$, and thus the probe geometry factors are transformed to:

$$\gamma'_j = g_j e^{i(\xi_j + j\phi)} \tag{15}$$

The observed phases are then, using Equation 12 and the above,

$$\psi'_n = \chi_n + \xi_n + \pi + n\phi_r \tag{16}$$

for the reference coil and

$$\psi'_j = \chi_j + \xi_j + \pi + j\phi_h \tag{17}$$

for the harmonics coil. One usually constructs a reference coil such that $\xi_n = 0$, so we drop it in the following discussion. If we now use Equation 16 to solve for χ_n, we will in general be disappointed to see that it doesn't equal zero as promised. We can remedy this problem by making use of the transformation property of the harmonic phase under a rotation of angle α:

$$\chi_j \to \chi_j + j\alpha \tag{18}$$

We rotate our coordinate system such that

$$\chi_n = \psi'_n - \pi - n\phi_r + n\alpha = 0 \tag{19}$$

Using this equation to solve for α, we transform the χ_j according to Equation 18, and then use Equation 17 to obtain

$$\begin{aligned} \chi_j &= \psi'_j - \xi_j - \pi - j\phi_h + j\alpha \\ &= \psi'_j - \frac{j}{n}\psi'_n - \xi_j + \left(\frac{j}{n} - 1\right)\pi - j\left(\phi_h - \phi_r\right). \end{aligned} \tag{20}$$

3.3.1 Phase ambiguities

The situation is not as simple as Equation 20 may suggest. One may equally well rotate the coordinate system so that $\chi_n = 2m\pi$, where m is any integer. Each choice of m will result in a different value of α and hence a different set of χ_j:

$$\chi_j(m) = \chi_j(m=0) + \frac{2mj}{n}\pi \tag{21}$$

Inspection of the above equation shows that for a dipole magnet, there is no problem, whereas for a quadrupole magnet, one has a two-fold ambiguity. In a normal quadrupole, one expects to find χ_2 pointing upward; however, the quadrupole amplitude is also a maximum in the downward direction, and if we select that as the reference direction, the phases of all the odd-j harmonics will be flipped by π. To avoid this ambiguity, one should start the probe rotation at an angle such that when we choose $m = 0$, we get χ_2 pointing in the desired direction. This is usually accomplished by putting a visual reference mark on the probe body and orienting that mark in a standard direction during measurement setup.

Harmonics measurements of sextupole magnets require even greater care, because there is a three-fold ambiguity.

3.3.2 Polarity reversal, clockwise rotation, probe insertion reversal

Equation 20 needs to be modified if the polarity of the signal is reversed, or if the rotation direction is clockwise rather than counter-clockwise, or if the probe is inserted with the opposite orientation.

If the polarity of the signal is reversed, e.g., by flipping the power supply leads, then the observed Fourier phases (including the reference) will change by π. It can also happen that the signal cable on one of the harmonic coils or the reference coil can have an extra twist relative to the others. These effects can be expressed as:

$$\psi'_j \to \psi'_j + \pi \tag{22}$$

and

$$\psi'_n \to \psi'_n + \pi \tag{23}$$

If one rotates the probe clockwise, this is equivalent to replacing θ by $-\theta$, and the observed Fourier phases will be transformed to

$$\psi'_j \rightarrow -\psi'_j \tag{24}$$

and

$$\psi'_n \rightarrow -\psi'_n. \tag{25}$$

The probe phases, ϕ_h and ϕ_r, are defined assuming a standard insertion orientation into the magnet: suppose we put a reference mark on one end of the probe, and install it in the magnet so the reference mark is near the lead end of the magnet. We could also install the probe backwards, so that the reference mark is now at the non-lead end. If we do that, however, the probe phases will now be reversed, so we will now get

$$(\phi_h - \phi_r) \rightarrow -(\phi_h - \phi_r). \tag{26}$$

One may apply a combination of some or all of the above, e.g., polarity reversal and clockwise rotation. The general expression for the phase is then a modification of Equation 20:

$$\chi_j = c_1\psi_j + c_2\pi - \frac{j}{n}(c_1\psi_n + c_3\pi) - c_4 j(\phi_h - \phi_r) - \xi_j + \frac{2mj\pi}{n}, \tag{27}$$

where c_1 is $+1$ for ccw rotation and -1 for cw; c_2 is -1 for normal polarity for the harmonics coil and 0 for reversed; c_3 is -1 for normal polarity for the reference coil and 0 for reversed; and c_4 is $+1$ for standard probe insertion and -1 for reversed.

In practice, one should adopt a standard way of doing the measurement and stick to it.

4 Morgan coils

Up to this point the discussion has been independent of specific probe geometry. We now consider the calculation of the γ_j factors for the commonly used probe styles.

The most common style is illustrated in Figure 1a. This is known as a Morgan coil[4]. For a coil of order m, there are $2m$ vertices with alternating directions ε_k equally spaced about the cylinder. Thus the spacing between adjacent vertices is $\phi = \pi/m$, and the probe geometry factor is given by

$$\gamma_j = \sum_{k=0}^{2m-1} (-1)^k \, e^{ikj\pi/m} \tag{28}$$

The summation reduces to a very simple form:

$$\gamma_j = \begin{cases} 2m, & \text{if } j = (2n-1)m, \; n = 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

Note that the γ_j are all real, and therefore all the $\xi_j = 0$. In practice, one generally uses a Morgan coil of order m to measure the reference amplitude of a magnet with symmetry m. One can also use an assortment of Morgan coils of various orders, all wound on the same probe cylinder, to collect a reasonably complete set of harmonics.

5 Tangential coils

Figure 1b illustrates a second type of commonly used coil called a tangential coil. It has two vertices which, unlike the Morgan coil, are asymmetrically located with respect to the cylinder axis. For the coil shown in the figure, the geometry factor is given by

$$\gamma_j = e^{i(\pi/2+\theta)j} - e^{i(\pi/2-\theta)j} = 2i^{j+1} \sin j\theta \tag{30}$$

Unlike the Morgan coil, the γ_j have nonvanishing values for most j, provided θ is suitably chosen. This means that the harmonics can be measured with a single tangential coil, rather than an assortment of Morgan coils.

5.1 Bucked tangential coils

The usefulness of tangential coils are improved by bucking them against a Morgan coil which is sensitive to the dominant harmonic. In Figure 1c we show the arrangement for a tangential coil with N turns bucked against a dipole ("belly band") Morgan coil having M turns. By comparing Equations 29 and 30, we see that the bucking condition will be satisfied ($\gamma_1 = 0$)

if we choose θ such that $M = N\sin\theta$. In that case the geometry factors become

$$\gamma_j = 2i^{j+1}\sin j\theta + \frac{M}{N}\left(1-(-1)^j\right) \tag{31}$$

To obtain the harmonic amplitudes (per Equation 14) and phases (per Equation 20), we need to explicitly calculate g_j and ξ_j. Inspection of the above shows that

- if j is odd:

$$g_j = 2\left[(-1)^{(j+1)/2}\sin j\theta + M/N\right] \tag{32}$$

$$\xi_j = 0 \tag{33}$$

- if j is even:

$$g_j = 2\sin\ j\theta \tag{34}$$

$$\xi_j = (j+1)\pi/2 \tag{35}$$

However, since the amplitude is required to be positive, if $g_j < 0$ according to the above, then we set $g_j \to |g_j|$ and $\xi_j \to \xi_j + \pi$.

5.2 Quadrupole-bucked tangential coils

The coil arrangement in the previous section is useful only in dipole magnets; to use a tangential coil for measuring quadrupole magnets, one should buck against a 4-pole Morgan coil ($m = 2$). Figure 1d shows an example of quadrupole bucking. An N-turn tangential coil, centered about 45° is bucked with an M-turn 4-pole Morgan coil. Considered separately, the geometry factors are

$$\gamma_j = 2ie^{ij\pi/4}\sin j\theta \tag{36}$$

for the tangential coil and

$$\gamma'_j = \begin{cases} 4, & j = 2(2n-1),\ n = 1,2,3,\ldots \\ 0, & \text{otherwise} \end{cases} \tag{37}$$

for the Morgan coil. The bucking condition (cancellation of the quadrupole term) will be satisfied if

$$N\gamma_2 + M\gamma'_2 = 0 \tag{38}$$

This establishes the requirement for the separation angle θ:

$$\sin 2\theta = 2M/N \tag{39}$$

As an example, if we choose $N = 4M$, then we see that $\theta = 15°$.

The total geometry factor is calculated using

$$\gamma_j = \gamma_j(\text{tangential}) + \frac{M}{N}\gamma_j(\text{Morgan}) \tag{40}$$

- Case $j = 2(2n-1)$:

$$\begin{aligned} \gamma_j &= 2e^{i(j+2)\pi/4}\sin j\theta + 4M/N \\ &= (-1)^n 2\sin j\theta + 4M/N \end{aligned} \tag{41}$$

  We see that $g_j = \gamma_j$ and $\xi_j = 0$, unless $\gamma_j < 0$, in which case we let $g_j = |\gamma_j|$ and $\xi_j = \pi$.

- Other j:

$$\gamma_j = 2e^{i(j+2)\pi/4}\sin j\theta \tag{42}$$

$$g_j = 2\sin j\theta \tag{43}$$

$$\xi_j = (j+2)\pi/4 \tag{44}$$

  If $g_j < 0$, then substitute $g_j \to |g_j|$ and $\xi_j \to \xi_j + \pi$.

It has been suggested[5] that it is advisable to buck out the dipole component as well because of probe motion imperfections. In that case, one will have to construct an assembly of more than one bucking coil. As of this writing, no quadrupole-bucked tangential coil has been built for use at MTF.

References

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